Light hadron production in $B_c \to J/\psi + X$ decays

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Decays of ground state B_c-meson $B_c \to J/\psi + n\pi$ are considered. Using existing parametrizations for $B_c \to J/\psi$ form-factors and $W^* \to n\pi$ spectral functions we calculate branching fractions and transferred momentum distributions of $B_c \to J/\psi + n\pi$ decays for $n = 1, 2, 3, 4$. Inclusive decays $B_c \to J/\psi + \bar{u}d$ and polarization asymmetries of final charmonium are also investigated. Presented in our article results can be used to study form-factors of $B_c \to J/\psi$ transitions, π-meson system spectral functions and give the opportunity to check the factorization theorem.

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I. INTRODUCTION

Recent measurements of B_c-meson mass and lifetime in CDF [1] and D0 [2] experiments allow us to hope that more detailed investigation of this particle on LHC collider, where about 10^{10} B_c-events per year are expected, would clarify mechanisms of B_c production and decay modes. Currently only products of B_c-meson production cross section and branching fractions of decays $B_c \to J/\psi\pi$, $J/\psi\ell\nu$ are known experimentally. For example, the following ratios are measured [3]:

$$\frac{\sigma_{B_c} Br\left(B_c \to J/\psi e^+\nu_e\right)}{\sigma_B Br\left(B_c \to J/\psi K\right)} = 0.282 \pm 0.038 \pm 0.074$$

for positron in the final state and

$$\frac{\sigma_{B_c} Br\left(B_c \to J/\psi \mu^+\nu_\mu\right)}{\sigma_B Br\left(B_c \to J/\psi K\right)} = 0.249 \pm 0.045^{+0.107}_{-0.076}$$

for muon. These ratios are about an order of magnitude higher than the theoretical predictions based on current estimates of B_c-meson production cross section and branching fraction $Br(B_c \to J/\psi\ell\nu) \approx 2\%$ [4]. The mode $B_c \to J/\psi\pi$ was used mainly to determine precisely B_c-meson mass. No information on production cross section, decay branching fraction, and even the product of these quantities was determined in this experiment.

Investigation of other B_c-meson decay channels and determination of their branching fractions will be one of interesting tasks of future experiments on LHC. Weak B_c decays can be caused by decays of both constituent quarks. Dominant are c-quark decay modes, which amount to $\sim 70\%$ of all B_c-meson decays. Unfortunately, none of such reactions were observed, although large branching fractions are expected for some of these decay modes (for example, for $B_c \to B_s\rho$ we have approximately 16% branching fraction). Mentioned above decays $B_c \to J/\psi\ell\nu$ and $B_c \to J/\psi\pi$ are examples of other class, caused by b-quark decay. Total branching fraction of this process is about 20%.

In the present paper we will fill the gap in existing theoretical predictions of B_c-meson decay branching fractions [4–7] and consider multi-particle processes $B_c \to J/\psi + n\pi$ with $n = 1, 2, 3, 4$. These reactions are caused by weak b-quark decay $b \to cW^* \to c\bar{u}d$ and clean analogy with similar τ-lepton decays ($\tau \to \nu_\tau + n\pi$) can be easily seen. This analogy allows us to use existing experimental data on τ-lepton decays and give reliable predictions of $B_c \to J/\psi + n\pi$ branching fractions.

In the next section we give analytical expressions for distributions of $B_c \to J/\psi + n\pi$ decays branching fractions over invariant mass of the light hadron system and study different asymmetries of final J/ψ-meson polarization as a function of this kinematic variable. In section III we use existing experimental data on τ-lepton decays calculate branching fractions of $B_c \to J/\psi + n\pi$ decays for $n = 1, 2, 3, 4$. In section IV inclusive reaction $B_c \to J/\psi\bar{u}d$ is considered in connection with duality relation. Short results of our work are given in the final section.

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II. ANALYTIC RESULTS

B_c-meson decays into light hadrons with vector charmonium J/ψ production are caused by b-quark decay $b \to W^* \to c\bar{u}d$ (see diagram shown in fig.1). The effective lagrangian of the latter process reads

$$\mathcal{H}_{\rm eff} \;=\; \frac{G_F}{2\sqrt{2}} V_{cb} V^*_{ud} \left[C_+(\mu) O_+ + C_-(\mu) O_-\right],$$

where G_F is Fermi coupling constant, V_{ij} are the elements of CKM mixing matrix, $C_\pm(\mu)$ are Wilson coefficients, that take into account higher QCD corrections and operators $O_\pm$ are defined according to

$$O_\pm \;=\; (\bar{d}_i u_j)_{V-A} (\bar{c}_i b_j)_{V-A} \pm (\bar{d}_j u_i)_{V-A} (\bar{c}_i b_j)_{V-A}.$$

In this expression i, j are color indexes of quarks and $(\bar{q}_1 q_2)_{V-A} = \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$. Since in our decays light quark pair should be in color-singlet state, the amplitude of the considered here processes is proportional to

$$a_1(\mu) \;=\; \frac{1}{2N_c} \left[(N_c - 1) C_+(\mu) + (N_c - 1) C_-(\mu)\right]$$

If QCD corrections are neglected, one should set $a_1(\mu) = 1$. Leading logarithmic strong corrections lead to dependence of this coefficient on the renormalization scale μ [8], and on $\mu \sim m_b$ it is equal to

$$a_1\left(m_b\right) \;=\; 1.17.$$

The matrix element of the decay $B_c \to J/\psi + \mathcal{R}$, where $\mathcal{R}$ is some set of light hadrons, has the form

$$\mathcal{M}\left[B_c \to W^* J/\psi \to \mathcal{R} J/\psi\right] \;=\; \frac{G_F V_{cb}}{\sqrt{2}} a_1 \mathcal{H}^\mu \epsilon^{\mathcal{R}}_\mu. \tag{1}$$

In this expression $\epsilon^{\mathcal{R}}$ is the effective polarization vector of virtual W-boson and

$$\mathcal{H}_\mu \;=\; \langle J/\psi \left| \bar{c} \gamma_\mu \left(1-\gamma_5\right) b \right| B_c \rangle = \mathcal{V}_\mu - \mathcal{A}_\mu.$$

Vector and axial currents are equal to

$$\begin{aligned}
\mathcal{V}_\mu &= \langle J/\psi \left| \bar{c} \gamma_\mu b \right| B_c \rangle = i \epsilon^{\mu\nu\alpha\beta} \epsilon^\psi_\nu \left(p+k\right)_\alpha q_\beta F_V\left(q^2\right), \\
\mathcal{A}_\mu &= \langle J/\psi \left| \bar{c} \gamma_\mu \gamma_5 b \right| B_c \rangle = \epsilon^\psi_\mu F^A_0\left(q^2\right) + \left(\epsilon^\psi p\right) \left(p+k\right)_\mu F^A_+\left(q^2\right) + \left(\epsilon^\psi p\right) q_\mu F^A_-\left(q^2\right),
\end{aligned}$$

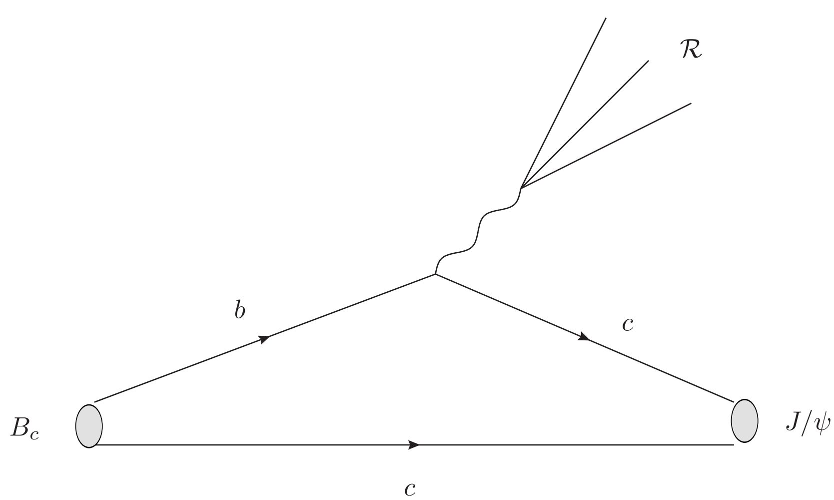

Figure 1: $B_c \to J/\psi + \mathcal{R}$

		SR	QM	LC
F_V	$F_V(0)$, GeV^{-1}	0.11	0.10	0.08
	M_{pole}, GeV	4.5	4.5	4.5
F_0^A	$F_0^A(0)$, GeV	5.9	6.2	4.7
	M_{pole}, GeV	4.5	4.5	6.4
F_+^A	$F_+^A(0)$, GeV^{-1}	-0.074	-0.70	-0.047
	M_{pole}, GeV	4.5	4.5	5.9

Table I: Parameters of B_c-meson form-factors

where p and k are the momenta of B_c- and J/ψ-mesons, $q = p - k$ is the momentum of virtual W-boson, and $F_V(q^2)$, $F^A_{0,\pm}(q^2)$ are form-factors of $B_c \to J/\psi W^*$ decay. Due to vector current conservation and partial axial current conservation the contribution of the form-factor F^A_- are suppressed by small factor $\sim (m_u + m_d)^2/M^2_{B_c}$, so we will neglect it in the following.

One can use different approaches when deriving the form of the form-factors $F(q^2)$. First of all, it is clear, that quark velocity in heavy quarkonia is small in comparison with c, so one can describe heavy quarkonia in the terms of non-relativistic wave-functions. This fact was used on the so called Quark Models [4, 5, 9–14]. In the following we will refer to this set of form-factors as *QM*.The speed of the final charmonium in B_c-meson rest frame, on the other hand, is large, so one can expand the amplitude of the considered here process in the powers of small parameter $M_{J/\psi}/M_{B_c}$, as it was done in papers [15–20]. In what follows, we will refer to this set of form-factors as *LC*. One can also use 3-point QCD sum rules to obtain the information on $B_c \to J/\psi W^*$form-factors [4, 11, 21, 22] (*SR*).

In our paper we use the following simple parametrization of form-factors

$$F\left(q^2\right) = \frac{F(0)}{1 - q^2/M^2_{pole}},$$

where numerical values of parameters $F_i(0)$ and M_{pole} are presented in table I.

The width of the $B_c \to J/\psi\mathcal{R}$ decay is

$$d\Gamma\left(B_c \to J/\psi\mathcal{R}\right) = \frac{1}{2M}\frac{G_F^2 V_{cb}^2}{2} a_1^2 \mathcal{H}^\mu \mathcal{H}^{*\nu} \epsilon_\mu \epsilon_\nu^{*\mathcal{R}} d\Phi\left(B_c \to J/\psi\mathcal{R}\right),$$

where Lorentz-invariant phase space is defined according to

$$d\Phi\left(Q \to p_1 \dots p_n\right) = (2\pi)^4 \delta^4\left(Q - \sum p_i\right) \prod \frac{d^3 p_i}{2E_i (2\pi)^3}.$$

It is well known, that the following recurrent expression holds for this phase space:

$$d\Phi\left(B_c \to J/\psi\mathcal{R}\right) = \frac{dq^2}{2\pi} d\Phi\left(B_c \to J/\psi W^*\right) d\Phi\left(W^* \to \mathcal{R}\right).$$

Using this expression one can perform the integration over phase space of the final state $\mathcal{R}$:

$$\frac{1}{2\pi}\int d\Phi\left(W^* \to \mathcal{R}\right) \epsilon_\mu^{\mathcal{R}} \epsilon_\nu^{\mathcal{R}*} = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right) \rho_T^{\mathcal{R}}\left(q^2\right) + q_\mu q_\nu \rho_L^{\mathcal{R}}\left(q^2\right),$$

where spectral functions $\rho^{\mathcal{R}}_{T,L}\left(q^2\right)$ are universal and can be determined from theoretical and experimental analysis of some other processes, for example $\tau \to \nu_\tau \mathcal{R}$ decay or electron-positron annihilation $e^+e^- \to \mathcal{R}$. Due to vector current conservation and partial axial current conservation spectral function $\rho_L^{\mathcal{R}}$ is negligible on almost whole kinematical region, so we will neglect is in our paper. Explicit expressions for spectral function $\rho_T^{\mathcal{R}}$ for different final states $\mathcal{R}$ are given in the next section.

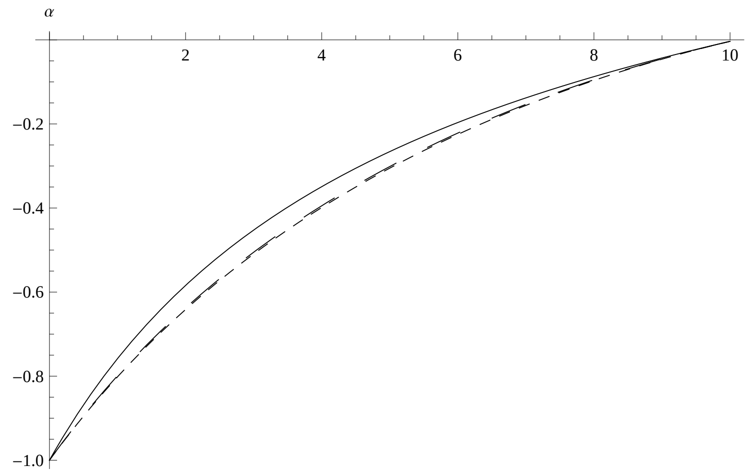

Figure 2: Polarization asymmetry α of final J/ψ-meson in $B_c \to J/\psi + \mathcal{R}$ decays as a function of squared transferred momentum q^2 (in GeV^2). Solid, dashed and dot-dashed lines stand for SR [7, 11], QM [4], and LC [20] respectively

Differential distributions of longitudinally and transversely polarized J/ψ-meson in $B_c \to J/\psi + \mathcal{R}$ decays can easily be obtained from presented above expressions. In the case of longitudinal polarization the polarization vector ϵ^ψ is equal to

$$\epsilon^\psi_\mu(\lambda = 0) \; = \; \frac{M}{2M_V}\left\{\beta, 0, 0, \frac{M^2 + M_V^2 - q^2}{M^2}\right\},$$

where z-axes is chosen in the direction of J/ψ movement, M and M_v are B_c- and J/ψ-meson masses and

$$\beta \; = \; \sqrt{\frac{(M + M_V)^2 - q^2}{M^2}}\sqrt{\frac{(M - M_V)^2 - q^2}{M^2}}.$$

Differential distribution has the form

$$\begin{aligned}\frac{d\Gamma\left[B_c \to J/\psi_{\lambda=0} + \mathcal{R}\right]}{dq^2} \; &= \; \frac{G_F^2 M^3 V_{cb}^2 a_1^2 \beta}{128\pi M_V^2}\rho_T^{\mathcal{R}}\left(q^2\right)\frac{M^4}{4M_V^2}\left\{\left(\beta^2 + \frac{4M_V^2 q^2}{M^4}\right)\left|F_0^A\right|^2 + M^4\beta^4\left|F_+^A\right|^2\right. \\ &+ \; \left. 2\beta^2\left(M^2 - M_V^2 - q^2\right)F_0^A F_+^A\right\}.\end{aligned}$$

In the case of transversely polarized vector meson ϵ^ψ_μ has the form

$$\epsilon^\psi_\mu(\lambda = \pm 1) \; = \; \left\{0, \frac{1}{\sqrt{2}}, \frac{\pm i}{\sqrt{2}}, 0\right\},$$

and the corresponding differential distribution is

$$\frac{d\Gamma\left[B_c \to J/\psi_{\lambda=\pm 1} + \mathcal{R}\right]}{dq^2} \; = \; \frac{G_F^2 V_{cb}^2}{32\pi M}a_1^2\beta q^2\rho_T^{\mathcal{R}}\left(q^2\right)\left\{\left|F_0^A\right|^2 + M^4\beta^2\left|F_V\right|^2 \pm \frac{2\beta M^2}{M_V^2}\mathrm{Re}\left(F_0^A F_V\right)\right\}.$$

It should be stressed, that the above expressions are universal and spectral function $\rho_T^{\mathcal{R}}(s)$ depends on the final state $\mathcal{R}$.

If the polarization if final vector meson is not observed, the q^2-distribution is, obviously,

$$\frac{d\Gamma\left[B_c \to J/\psi + \mathcal{R}\right]}{dq^2} \; = \; \sum_{\lambda=0,\pm 1}\frac{d\Gamma\left[B_c \to J/\psi_\lambda + \mathcal{R}\right]}{dq^2}. \tag{2}$$

It is also useful to study some polarization asymmetries. For example, polarization degree α is defined according to

$$\alpha \; = \; \frac{d\Gamma_{\lambda=+1} + d\Gamma_{\lambda=-1} - 2d\Gamma_{\lambda=0}}{d\Gamma_{\lambda=+1} + d\Gamma_{\lambda=-1} + 2d\Gamma_{\lambda=0}}.$$

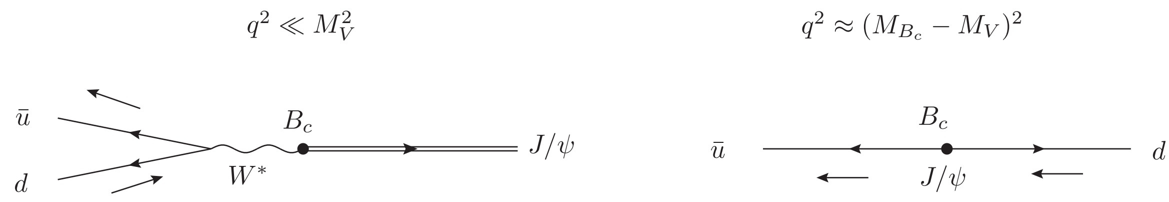

Figure 3: Kinematics of $B_c \to J/\psi u \bar{d}$ decay

Production of transversely polarized, longitudinally polarized and unpolarized J/ψ-meson corresponds to $\alpha = 1$, $\alpha = -1$ and $\alpha = 0$ respectively. We would like to note, that in the framework of factorization model this asymmetry does not depend on final state $\mathcal{R}$. So, experimental investigation of this asymmetry can be used for determination of B_c-meson form factors and test of QCD factorization. In fig.2 we show q^2-dependence of this asymmetry for different sets of B_c-meson form-factors. One can easily explain qualitatively the behavior of these curves. Let us consider q^2-dependence of asymmetry α in $B_c \to J/\psi \bar{u} d$ decay. At low q^2 the direction of $\bar{u}$- and d-quarks momenta in B_c-meson rest frame will be close to each other and opposite to the direction of the momentum of J/ψ-meson. The spin of light $\bar{u}$-antiquark (d-quark) is directed along (opposite to) its momentum (see fig.3a), so quark-antiquark pair has $\lambda = 0$ projection on Oz axis. From angular momentum conservation it follows, that J/ψ-meson should also be longitudinally polarized. This can be observed in figure 2, where at low q^2 we have $\alpha = -1$ for all sets of B_c-meson form-factors. In high q^2-region, on the contrary, direction of quark and antiquark momenta are opposite to each other and J/ψ-meson stay at rest in B_c-meson rest frame (see fig.3b). As a result, final J/ψ-meson is unpolarized in this region and $\alpha = 0$.

Another example is transverse asymmetry

$$\alpha_T = \frac{d\Gamma_{\lambda=1} - d\Gamma_{\lambda=-1}}{d\Gamma}.$$

This asymmetry also depends only on B_c-meson form-factors and its dependence on squared transferred momentum is shown in fig.4.

III. EXCLUSIVE DECAYS

In this section we present differential widths and branching fractions of the decays $B_c \to J/\psi + n\pi$ using presented above universal formula (2) and specific expressions for spectral function $\rho_T^{\mathcal{R}}\left(q^2\right)$.

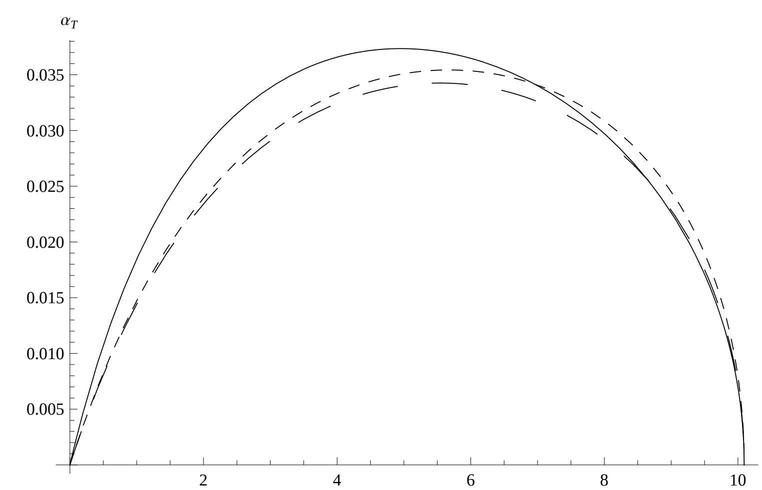

Figure 4: Transverse polarization asymmetry α_T of final J/ψ-meson in $B_c \to J/\psi + \mathcal{R}$ decays as a function of squared transferred momentum q^2 (in GeV^2). Solid, dashed and dot-dashed lines stand for SR [7, 11], QM [4], and LC [20] respectively

A. $B_c \to J/\psi\pi$

Let us first of all consider two-particle decays $B_c \to J/\psi\pi$ and $B_c \to J/\psi\rho$.

In the case of $B_c \to J/\psi\pi$ decay the $W^* \to \pi$ transition is expressed through leptonic constant f_π:

$$\langle \pi \left| \bar{u}\gamma_\mu\gamma_5 d \right| 0 \rangle = \sqrt{2} f_\pi q_\mu . \tag{3}$$

The numerical value of this constant can be determined from $\pi \to \mu\nu_\mu$ decay width: $f_\pi \approx 140$ MeV. The spectral function, that corresponds to vertex (3) is

$$\rho_T^\pi \left(q^2\right) = 2 f_\pi^2 \delta\left(q^2\right).$$

Using this spectral function it is easy to obtain the following values of $B_c \to J/\psi\pi$ decay branching fractions for different sets of form-factors:

$$\begin{aligned}
\mathrm{Br}_{LC}\left(B_c \to J/\psi\pi\right) &= 0.13\%, \\
\mathrm{Br}_{QM}\left(B_c \to J/\psi\pi\right) &= 0.17\%, \\
\mathrm{Br}_{SR}\left(B_c \to J/\psi\pi\right) &= 0.17\%.
\end{aligned}$$

B. $B_c \to J/\psi + 2\pi$

The 2π channel is saturated mainly by $B_c \to J/\psi\rho$ decay. The $W^* \to \rho$ transition vertex is also expressed through ρ-meson leptonic constant

$$\langle \rho \left| \bar{u}\gamma_\mu d \right| 0 \rangle = \sqrt{2} f_\rho M_\rho \epsilon_\mu$$

where $f_\rho \approx 150\,\mathrm{MeV}$. If one neglects the width of ρ-meson, the corresponding spectral function has the form

$$\rho_T^\rho \left(q^2\right) = 2 f_\rho^2 \delta\left(q^2 - m_\rho^2\right). \tag{4}$$

The branching fractions of $B_c \to J/\psi\rho$ for different sets of form-factors are:

$$\begin{aligned}
\mathrm{Br}_{LC}\left(B_c \to J/\psi\rho\right) &= 0.38\%, \\
\mathrm{Br}_{QM}\left(B_c \to J/\psi\rho\right) &= 0.44\%, \\
\mathrm{Br}_{SR}\left(B_c \to J/\psi\rho\right) &= 0.48\%.
\end{aligned}$$

In order to take ρ-meson width into account, one can use experimental data on $\tau \to \nu_\tau + 2\pi$ decay. The differential branching ratio of this reaction is equal to

$$\frac{d\Gamma\left(\tau \to \nu_\tau \mathcal{R}\right)}{dq^2} = \frac{G_F^2}{16\pi m_\tau} \frac{\left(m_\tau^2 - q^2\right)^2}{m_\tau^3} \left(m_\tau^2 + 2q^2\right) \rho_T^{\mathcal{R}}\left(q^2\right).$$

This method was used by ALEPH collaboration to measure the spectral function $\rho_T^{2\pi}(q^2)$ in the kinematically allowed region $q^2 < m_\tau^2$ [23] and can be approximated by the expression (see fig.5a)

$$\rho_T^{2\pi}(s) \approx 1.35 \times 10^{-3} \left(\frac{s - 4m_\pi^2}{s}\right)^2 \frac{1 + 0.64s}{\left(s - 0.57\right)^2 + 0.013},$$

where s is measured in GeV^2. In fig.5b we show corresponding distributions $d\Gamma\left(B_c \to J/\psi + 2\pi\right)/dq^2$. Solid, dashed and dash-dotted lines in this figure correspond to form-factors SR, QM, and LC respectively. The branching fractions of the decay $B_c \to J/\psi + 2\pi$ are almost equal to $B_c \to J/\psi\rho$ decay branching fractions:

$$\begin{aligned}
\mathrm{Br}_{LC}\left(B_c \to J/\psi\pi\pi\right) &= 0.35\%, \\
\mathrm{Br}_{QM}\left(B_c \to J/\psi\pi\pi\right) &= 0.44\%, \\
\mathrm{Br}_{SR}\left(B_c \to J/\psi\pi\pi\right) &= 0.48\%.
\end{aligned}$$

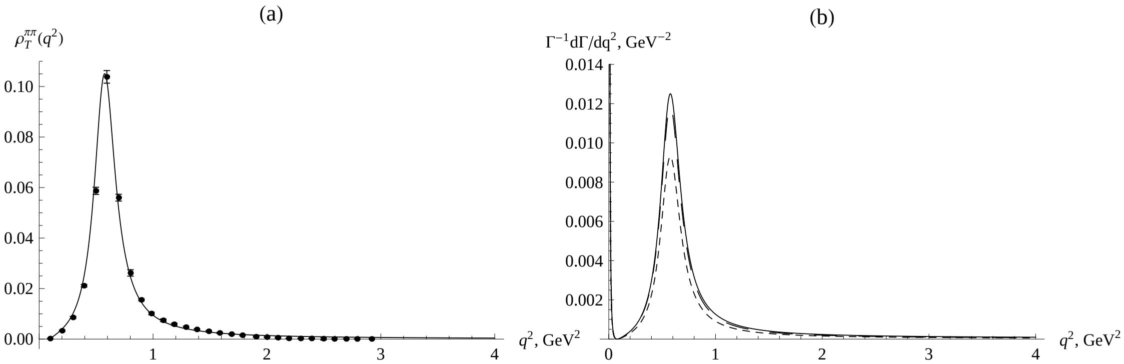

Figure 5: fig (a) — spectral function $\rho_T^{2\pi}$; (b) — $\Gamma^{-1}d\Gamma(B_c \to J/\psi + 2\pi)/dq^2$ distribution for different sets of B_c-meson form-factors. Solid, dashed and dot-dashed lines stand for SR [7, 11], QM [4], and LC [20] respectively

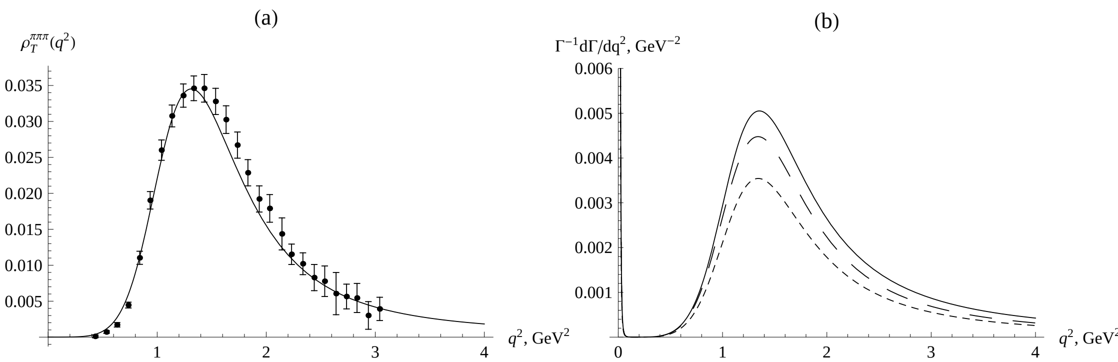

Figure 6: Spectral function and differential width for $B_c \to J/\psi + 3\pi$ decay. Notations are the same as in fig.5.

C. $B_c \to J/\psi + 3\pi$

In the case of $B_c \to J/\psi + 3\pi$ decay (where 3π stands for the sum of $\pi^-\pi^0\pi^0$ and $\pi^-\pi^+\pi^-$ decay modes) the G-parity of the final state is negative. So we can expect, that this mode is saturated by axial-vector resonance a_1. The width of this state is too large to neglect it, so we cannot use the expression similar to (4) for $W^* \to 3\pi$ transition. The corresponding spectral function can be determined from experimental and theoretical data on $\tau \to \nu_\tau + 3\pi$ decay. In our article we use the following expression to approximate this function (see. fig.6a):

$$\rho_T^{3\pi}(s) \approx 5.86 \times 10^{-5} \left(\frac{s - 9m_\pi^2}{s}\right)^4 \frac{1 + 190.s}{\left[(s-1.06)^2 + 0.48\right]^2}.$$

Distributions over q^2 for different sets of B_c-meson form factors are shown in fig.6b. The branching fractions of $B_c \to J/\psi + 3\pi$ decay are

$$\begin{aligned} \mathrm{Br}_{LC}(B_c \to J/\psi + 3\pi) &= 0.52\%, \\ \mathrm{Br}_{QM}(B_c \to J/\psi + 3\pi) &= 0.64\%, \\ \mathrm{Br}_{SR}(B_c \to J/\psi + 3\pi) &= 0.77\%. \end{aligned}$$

D. $B_c \to J/\psi + 4\pi$

In the decay $B_c \to J/\psi + 4\pi$ both $\pi^-\pi^0\pi^0\pi^0$ and $\pi^-\pi^+\pi^-\pi^-$ modes are possible in the following we consider the sum of these states. The kinematically allowed region in $\tau \to \nu_\tau + 4\pi$ decay is too small to determine the form of spectral function $\rho_T^{4\pi}$, so it is more convenient to use energy dependence of 4π production cross section in

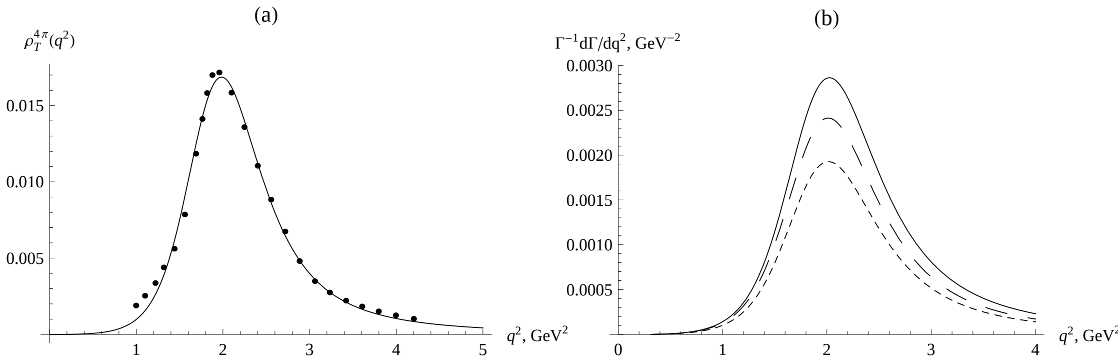

Figure 7: Spectral function and differential width for $B_c \to J/\psi + 4\pi$ decay. Notations are the same as in fig.5

electron-positron annihilation. It is easy to obtain the following expression for this cross section:

$$\sigma\left(e^+e^- \to 4\pi\right) = \frac{4\pi\alpha^2}{s}\rho_T^{4\pi}(s).$$

Spectral function $\rho_T^{4\pi}$, calculated from experimental data [24] is shown in fig.7a and later we use the following parametrization:

$$\rho_T^{4\pi}(s) \approx 1.8 \times 10^{-4}\left(\frac{s-16m_\pi^2}{2}\right)\frac{1+5.07s+8.63s^2}{\left[(s-1.83)^2+0.61\right]^2}.$$

The distributions corresponding to this spectral function are shown in fig.7b. The branching fraction for different sets of B_c-meson form-factors are

$$\begin{aligned}
\mathrm{Br}_{LC}\left(B_c \to J/\psi + 4\pi\right) &= 0.26\%,\\
\mathrm{Br}_{QM}\left(B_c \to J/\psi + 4\pi\right) &= 0.33\%,\\
\mathrm{Br}_{SR}\left(B_c \to J/\psi + 4\pi\right) &= 0.40\%.
\end{aligned}$$

IV. INCLUSIVE DECAYS AND DUALITY RELATION

Let us know consider the inclusive decay $B_c \to B_c + X$ where X stands for an arbitrary state of light hadrons. On quark level this reaction corresponds to $B_c \to J/\psi + \bar{u}d$ decay. If one neglects u- and d-quark masses, the spectral function of $W^* \to \bar{u}d$ transition is energy independent and equals to

$$\rho_T^{ud} = \frac{1}{2\pi^2}.$$

In fig.8 distributions of $B_c \to J/\psi + \bar{u}d$ decay branching fractions for different sets of B_c-meson form-factors are shown. Integrated branching fractions of this decay are

$$\begin{aligned}
\mathrm{Br}_{LC}\left(B_c \to J/\psi + \bar{u}d\right) &= 7\%,\\
\mathrm{Br}_{QM}\left(B_c \to J/\psi + \bar{u}d\right) &= 8.6\%,\\
\mathrm{Br}_{SR}\left(B_c \to J/\psi + \bar{u}d\right) &= 12\%.
\end{aligned}$$

It should be noted, that sum of presented above branching fractions (that is $Br(B_c \to J/\psi + n\pi)$, $n = 1, \dots 4$) gives only about 30% the inclusive decay branching fraction. So one could expect noticeable events with multi-pion production in B_c-meson decays.

Below KK-production threshold only π-mesons can be produced in $\bar{u}d$-pair hadronization, so the duality relation should be satisfied

$$\int^{(2m_K+\Delta)^2} \frac{1}{\Gamma}\frac{d\Gamma(B_c \to J/\psi\bar{u}d)}{dq^2} = \sum_n \mathrm{Br}(B_c \to J/\psi + n\pi),$$

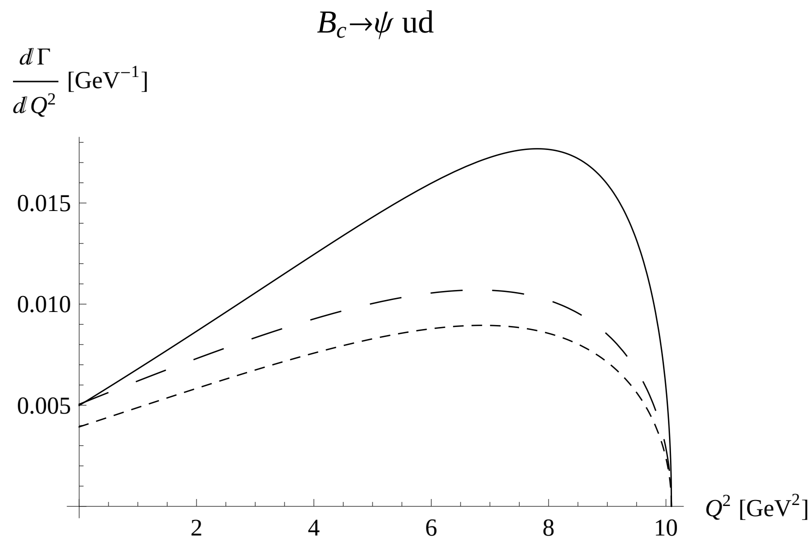

Figure 8: Differential $B_c \to J/\psi\bar{u}d$ branching fractions for different sets of B_c-meson form-factors. Notations are the same as in fig.5

	π	2π	3π	4π	$\bar{u}d$
LC	0.13	0.35	0.52	0.26	7
QM	0.17	0.44	0.64	0.33	8.6
SR	0.17	0.48	0.77	0.40	12

Table II: $B_c \to J/\psi\mathcal{R}$ decays branching fractions (in %) for different sets of B_c-meson form-factors

where Δ is the duality window. If we restrict ourselves to $n \leq 4$ in the right-hand side of this relation, it is valid for

$$\Delta \approx 0.6\,\mathrm{GeV}.$$

It is interesting to note that this value is almost independent on the choice of B_c-meson form-factors and close to the value of duality parameter in $gg \to J/\psi c\bar{c}$ and $\chi_b \to J/\psi c\bar{c}$ reactions [25, 26].

V. CONCLUSION

In our paper we study exclusive and inclusive decays of B_c-meson into light hadrons and vector charmonium J/ψ, that is the processes $B_c \to J/\psi + \bar{u}d$ and $B_c \to J/\psi + n\pi$ where $n = 1, 2, 3, 4$. According to QCD factorization theorem the amplitude of these processes splits into two independent parts. The first factor describes the decay $B_c \to J/\psi W^*$ and one can use existing parametrizations of B_c-meson form-factors to calculate this amplitude. The second factor describes the fragmentation of virtual W-boson. The information about these processes was taken from experimental distributions of multi-pion production in τ-lepton decays and electron-positron annihilation.

Our results are gathered in table II, where branching fractions of multi-pion production in $B_c \to J/\psi + n\pi$ for different B_c-meson form-factors are presented. The last column of this table contains the branching fraction of the inclusive decay $B_c \to J/\psi + \bar{u}d$. It is clear that up to KK-production threshold only π-mesons could be produced in $B_c \to J/\psi + X$ decay, so some duality relation should hold. In our article it is shown, that to satisfy this relation it is sufficient to integrate the inclusive spectrum up to squared transferred momentum $q^2 = (2m_K + \Delta)^2$. It turns out, that Δ is almost independent on the choice of B_c-meson form-factors and equals to ~ 0.6 GeV.

The other interesting point are the polarization asymmetries of final J/ψ-meson. In the framework of factorization model these asymmetries do not depend on the final state $\mathcal{R}$, so one can use them to investigate form-factors of B_c-meson and to test the factorization theorem. In our paper we present the polarization degree $\alpha = (d\Gamma_T/dq^2 - 2d\Gamma_L/dq^2)/(d\Gamma_t/dq^2 + 2d\Gamma_L/dq^2)$ and transverse polarization asymmetry $\alpha_T = (d\Gamma_{\lambda=1}/dq^2 - d\Gamma_{\lambda=-1}/dq^2)/(d\Gamma/dq^2)$ for different sets of form-factors.

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